Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level in Pure Mathematics P1 (WMA11) Paper 01

Question Number	Scheme	Marks
1.	$\int\left(\frac{8}{3}x^3-\frac{1}{2}x^{-\frac{1}{2}}-5\right)\mathrm{d}x=\frac{8}{3}\times\frac{x^4}{4}-\frac{1}{2}\times 2x^{\frac{1}{2}}-5x+c$	M1 A1
	$=\frac{2}{3}x^4-x^{\frac{1}{2}}-5x+c$	A1 A1
		(4 marks)

M1 For raising a power of x by 1 seen at least once (i.e. $x^n\to x^{n+1}$) or $-5\to-5x$.
The index does not have to be processed for this mark.
Award following incorrect manipulations eg $-\frac{1}{2\sqrt{x}}\to ...x^{\frac{1}{2}}\to ...x^{\frac{3}{2}}$

A1 For two of $\frac{8}{3}\times\frac{x^4}{4}$, $-\frac{1}{2}\times 2x^{\frac{1}{2}}$, $-5x^1$ correct (unsimplified).
This may be implied by a correct simplified answer. May be seen on different lines. Indices must be processed. Ignore any spurious notation for any of the A marks.

A1 For two of $\frac{2}{3}x^4$, $-x^{\frac{1}{2}}$, $-5x$ correct and in simplest form. Accept forms such as $\frac{2x^4}{3}$ and $-\sqrt{x}$
Condone $-5x^1$, $-1x^{\frac{1}{2}}$ but not $-\frac{5x}{1}$. Correct terms may be seen on different lines. Allow $\frac{2}{3}$ to be written as $0.\dot{6}$ or 0.6666….. only (must be exact)

A1 Fully correct **on one line**, and simplified with $+c$. Accept simplified equivalents (see above)

Question Number	Scheme	Marks
2 (a)	$3^{3x} = \left(3^x\right)^3 = y^3$	B1
		(1)
(b)	$\frac{1}{3^{x-2}} = \frac{1}{3^x \times 3^{-2}} = \frac{9}{y}$	M1 A1
		(2)
(c)	$\frac{81}{9^{2-3x}} = \frac{9^2}{9^{2-3x}} = 9^{2-(2-3x)} = 9^{3x} = 3^{6x} = y^6$	M1 A1
		(2)
		(5 marks)

Note: Correct answer in any part implies full marks for that part

(a)

B1 y^3 Condone $(y)^3$. Ignore once correct answer seen.

(b)

M1 For correct application of the addition/subtraction law so award for eg:

$\frac{1}{3^x \times 3^{-2}}$ or $\frac{1}{3^x \div 3^2}$ or $\frac{1}{\left(\frac{3^x}{3^2}\right)}$ or $\frac{1}{y \times 3^{-2}}$ or $\frac{3^2}{3^x}$ or sight of 9 or $\frac{1}{y}$ oe

A1 For $\frac{9}{y}$ or $9y^{-1}$ but NOT expressions that still contain $\div$ or fractions within fractions or $3^2 y^{-1}$. Ignore once correct answer seen.

(c)

M1 For simplifying the indices to expressions of the form $9^{\cdots}$ or $3^{\cdots}$ (not as a denominator) so award for 9^{3x}, $\left(9^x\right)^3$, 3^{6x}, $\left(3^x\right)^6$, or $k \times y^6$, $k \neq 0$ which must come from correct working. Also allow unsimplified equivalent expressions of the final answer eg $\frac{1}{y^{-6}}$ as long as it in terms of y

A1 y^6 only (not eg $\frac{1}{y^{-6}}$). Ignore once correct answer seen. Condon $(y)^6$

Note: In all parts they may work from $y = 3^x$ and manipulate both sides to the given answer which is acceptable.

Eg part (b): $y = 3^x \Rightarrow y \times 3^{-2} = 3^{x-2}$ (M1) $\Rightarrow \frac{1}{y \times 3^{-2}} = \frac{1}{3^{x-2}} \Rightarrow \frac{9}{y} = \frac{1}{3^{x-2}}$ (A1)

Eg part (c): $y = 3^x \Rightarrow y^{-6} = 3^{-6x} \Rightarrow 81y^{-6} = 3^{4-6x}$ (M1) $\Rightarrow 81y^{-6} = 9^{2-3x} \Rightarrow \frac{81}{9^{2-3x}} = y^6$ (A1)

Question Number	Scheme	Marks
3.(a)	Attempts $\left(\frac{dy}{dx}=\right)$ $2x+3$ at $x=3$	M1
	At $x=3$ gradient of tangent = 9	A1
		(2)
(b)	$\left(y_Q=\right)$ $(3+h)^2+3(3+h)-2$	B1
	Gradient $PQ=\frac{(3+h)^2+3(3+h)-2-16}{3+h-3}$ $=\frac{9h+h^2}{h}=9+h$	M1 A1
		(3)
(c)	States as $h\to 0$ Gradient $PQ \to 9$ = Gradient of tangent	B1
		(1)
		(6 marks)

(a)

M1 Attempts to find the value of $\left(\frac{dy}{dx}=\right)$ $ax+3,\quad a>0$ at $x=3$. Look for 3 to be substituted into the expression and proceeding to a value.

A1 9 (Answer only scores both marks)

(b)

B1 $\left(y_Q=\right)$ $(3+h)^2+3(3+h)-2$ (seen or implied)

M1 Attempts $\pm\frac{y_Q-16}{x_Q-3}$ condoning slips, but must be a genuine attempt at y_Q. Condone lack of brackets if implied by later working.

A1 $9+h$ with no errors seen and not originating from methods in calculus. This expression may immediately follow a correct simplified expression of $y_Q=h^2+9h+16$

(c)

B1 States as $h\to 0$ Gradient $PQ\to 9$ = Gradient of tangent (oe)

They must have achieved $9+h$ in (b) and 9 in (a)

There should be reference to "limit" or "as *h* tends to 0" (words or symbols) and linked to part (a) (so same gradient, or showing the answers agree). But be generous with the explanation beyond these contraints.

Question Number	Scheme	Marks
4 (a)	States or uses $\cos AOD = \frac{4}{12} \Rightarrow$ angle $AOD = 1.231$ *	M1 A1*
		(2)
(b)	Attempts $\frac{1}{2}r^2\theta$ with $r = 12$ and $\theta = \pi \pm 1.231$ or 1.231	M1
	Attempts area $AOD = \frac{1}{2}\times"4"\times\sqrt{12^2 - "4"^2}$ oe (22.627...)	M1
	Attempts sector − triangle = $\frac{1}{2}\times 12^2 \times(\pi + 1.231) - \frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$	
	(314.8….) − (22.627…)	ddM1
	or	
	Attempts circle-sector-triangle	
	$\pi\times 12^2 - \frac{1}{2}\times 12^2\times(\pi - 1.231) - \frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$	
	452.38… − 137.562…. − 22.627……	
	= awrt 292.2 (m^2)	A1
		(4)
(c)	Attempts $s = r\theta$ with $r = 12$ and $\theta = \pi \pm 1.231$ or 1.231	M1
	Attempts $P = 16 + \sqrt{12^2 - 4^2} + 12(\pi + 1.231)$ oe	dM1
	= awrt 79.8 (m)	A1
		(3)
		(9 marks)
Alt(a)	$AD = \sqrt{12^2 - 4^2} = 8\sqrt{2}$ $\cos AOD = \frac{12^2 + 4^2 - (8\sqrt{2})^2}{2\times 12\times 4} \Rightarrow$ angle $AOD = \cos^{-1}\left(\frac{1}{3}\right) = 1.231$	M1A1*

(a)

M1 Attempts $\cos AOD = \frac{4}{12} \Rightarrow$ angle $AOD = ...$ via a correct method.

Alternatively attempts to find AD using Pythagoras' theorem correctly **and** uses the appropriate sine or tangent. They may attempt to use the cosine rule with all three side (see alt(a)).

Other candidates are finding angle *OAD* and using angles in a triangle to find angle *AOD*.

Do not award this mark for candidates who attempt to use 1.231 and compare with $\frac{4}{12}$

A1* Achieves angle $AOD = 1.231$ following a valid method and at least one step of working shown.

Minimum acceptable $\cos^{-1}\left(\frac{4}{12}\right) = 1.231$ or $\cos AOD = \frac{4}{12} \Rightarrow$ angle $AOD = 1.231$

Only withhold this mark if

- awrt 1.231 radians is not achieved following a correct method. FYI sight of 1.23095… is likely to imply both marks)
- they work in degrees but fail to achieve awrt 70.5° **before** proceeding to 1.231 radians

We are going to condone on this occasion any rounding that may be written as part of their intermediate steps as long as they achieve awrt 1.231. Ignore any omission of brackets.

(b)

M1 Attempts $\frac{1}{2}r^2\theta$ with $r=12$ and an allowable θ. ($\theta=\pi\pm1.231$ or 1.231).

$\angle AOC=$ awrt 1.911 which may appear as ($\pi-1.231$) or ($2\pi-(\pi+1.231)$) (minor sector)

$\angle AOC=$ awrt 4.373 which may appear as ($\pi+1.231$) or ($2\pi-(\pi-1.231)$) (major sector)

The embedded values in an expression is sufficient for this mark. (awrt 315, awrt 88.6 or awrt 138 also implies this mark. Condone candidates who round 1.231 or use eg awrt 4.4 or awrt 1.9 radians.

M1 Correct method to find area of triangle *AOD*. Eg $\frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$, $\frac{1}{2}\times"4"\times12\sin1.231$, awrt 22.6.

The angle may be in degrees (70.5….) and condone using awrt 1.2 radians.

Also allow an alternative method finding the area of the rectangle – area of trapezium. Allow errors in their method to finding "4" but *r* must be 12.

ddM1 Full method to find the correct area $\frac{1}{2}\times12^2\times(\pi+1.231)-\frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$ oe. The embedded values in an expression is sufficient for this mark and the angle may be awrt 4.4. It is dependent on the two previous method marks.

Alternatively they may find the area by

Area of circle – area of minor sector – area of triangle

$$\pi\times12^2-\frac{1}{2}\times12^2\times(\pi-1.231)-\frac{1}{2}\times"4"\times\sqrt{12^2-"4"^2}$$

A1 awrt 292.2 (m^2)

(c)

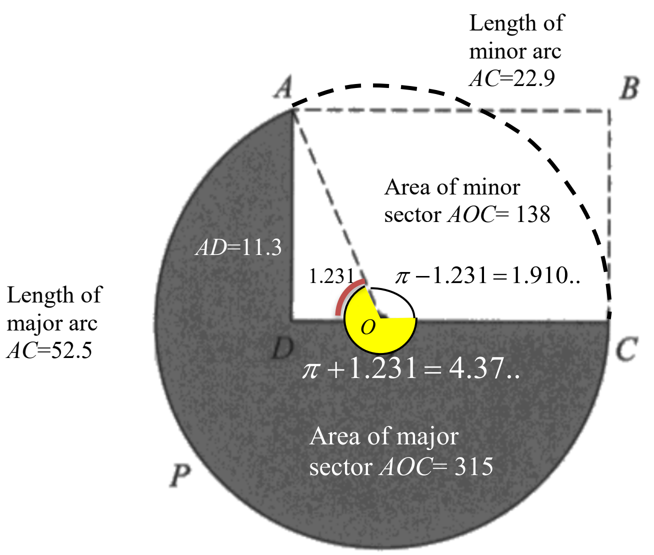
M1 Attempts $s=r\theta$ with $r=12$ and an allowable θ. ($\theta=\pi\pm1.231$ or 1.231 as above in part (b)). The embedded values in an expression is sufficient for this mark. (awrt 52.5 or awrt 14.8 or awrt 22.9 can imply this mark. Condone candidates who round 1.231 or use eg awrt 4.4 or awrt 1.9 radians).

dM1 Full method to find the perimeter $P=16+\sqrt{12^2-"4"^2}+12(\pi+1.231)$ oe. The embedded values in an expression is sufficient for this mark and the angle may be awrt 4.4 or awrt 1.9.

Alternatively they may find the perimeter *P* via circumference − minor arc + 16+"$8\sqrt{2}$"

$$P=16+\sqrt{12^2-"4"^2}+24\pi-12(\pi-1.231)$$

A1 awrt 79.8 (m)



Question Number	Scheme	Marks
5. (a)	$20x^3-50x^2-30x=0 \Rightarrow 10x\left(2x^2-5x-3\right)=0$	
	$\Rightarrow 10x\left(2x+1\right)\left(x-3\right)=0$	M1
	$\Rightarrow x=0,-\frac{1}{2},3$	A1, A1
		(3)
(b)	Sets or implies $\left(y+3\right)^{\frac{1}{2}}=0$ or $-\frac{1}{2}$ or 3	B1ft
	Full method to find y	M1
	$y=6$	A1ft,
	$y=-3,\ 6$	A1
		(4) **(7 marks)**

(a) **Note the question says using algebra so answers only scores 0 marks.**

M1 For attempting to factorise. They may do this by cancelling/factorising out $\ldots x$ to achieve an expression of the form $Dx(Ax^2+Bx+C)$ where $A,B,C\neq 0$ (but could be 1) and $D=1,2,5$ or 10
If they only have the resulting quadratic $\left(2x+1\right)\left(x-3\right)$ they must state somewhere that $x=0$ (usually as one of their answers)
Equally they may proceed straight to a fully factorised form eg. $x\left(2x+1\right)\left(x-3\right)\ \ \left(=0\right)$
We will condone on this occasion $x\left(x+\frac{1}{2}\right)(x-3)$

A1 Two of $x=0,-\frac{1}{2},3$ **following M1 awarded** .

A1 All of $x=0,-\frac{1}{2},3$ They do not have to be stated on one line together. Withhold if any additional solutions.

(b)

B1ft Sets $\left(y+3\right)^{\frac{1}{2}}=$ any of their solutions from (a) or this may be implied by later work on one of their solutions from (a). Note $(y+3)=0\Rightarrow y=-3$ is insufficient for the B mark but is acceptable for the final A mark (see below)

M1 Full method to find a value for y from a non-zero solution for x.
Scored for **squaring** and **subtracting 3**
Allow this to be scored for squaring and subtracting 3 from a negative value as well

A1ft $y=6$. Follow through on their positive solution from part (a)

A1 Both of $y = -3, 6$ with no incorrect working seen. If the solution $-\frac{11}{4}$ is found it must be discounted by the candidate (eg crossed out/rejected).

Beware that part (b) is hence so they must use their solutions from (a) and be careful regarding the -3 solution.

$(y+3)^{\frac{1}{2}} = 0 \Rightarrow y = -3$ and $(y+3) = 0 \Rightarrow y = -3$ is acceptable for the final A mark BUT

$(y+3)^{\frac{1}{2}} = 0 \Rightarrow y + 3 = \sqrt{0} \Rightarrow y = -3$ is an incorrect method so A0

Give benefit of the doubt where it is unclear but send to review if unsure.

Question Number	Scheme	Marks
6.(a)	Attempts to find the gradient of $3x-4y+20=0 \Rightarrow y=\underline{\underline{\frac{3}{4}}}x+5$	M1
	Equation l_2 is $y-0="\frac{3}{4}"(x-8) \Rightarrow 3x-4y-24=0$ oe	M1, A1
		(3)
(b)	$P=\left(-\frac{20}{3},0\right), Q=(0,5)$	B1
	Area $PQRS=$ $PR\times OQ=\left(8+"\frac{20}{3}"\right)\times"5"=\frac{220}{3}$	M1, A1
		(3)
(c)	$\overrightarrow{QR}=\begin{pmatrix}8\\-5\end{pmatrix} \Rightarrow S=\left(-\frac{20}{3}+8,0-5\right)=\left(\frac{4}{3},-5\right)$	M1, A1
		(2) **(8 marks)**
Alt(c)	Solve their $y="\frac{3}{4}x-6"$ with their $"y=-\frac{5}{8}\left(x+\frac{20}{3}\right)"$	
	$\frac{3}{4}x-6=-\frac{5}{8}\left(x+\frac{20}{3}\right) \Rightarrow \frac{11}{8}x=\frac{11}{6} \Rightarrow x=...$	M1
	$S=\left(\frac{4}{3},-5\right)$	A1
		(2)

If you see any unusual approaches which may involve vectors and matrices then please send to review if you are unsure how to mark.

(a)

M1 For an attempt to find the gradient of $3x-4y+20=0$
Look for an attempt to rearrange $3x-4y+20=0$ and make y the subject.
Expect to see $\pm 4y=...$ **followed by** $y=...$ **or equivalent** if they do not achieve the correct value.

M1 For using the same gradient as their gradient of l_1 with $R(8,0)$ to form a linear equation.
If they use $y=mx+c$ they must proceed as far as $c=...$ starting from a correct equation with their gradient of l_1 and the coordinates correctly substituted in.
Some candidates may set $3x-4y+c=0$ and substitute $(8,0)$ to find c which is M1M1.

A1 $3x-4y-24=0$ or any integer multiple. All terms must be on one side of an equation $=0$.
Correct equation implies full marks and it must be seen in (a) to score the A mark. (They may make errors in the manipulation of l_1 which is condoned).

Mark parts (b) and (c) together.

(b)

B1 $P=\left(-\frac{20}{3},0\right), Q=(0,5)$. This may be implied by subsequent calculations or seen on a diagram

M1 A full attempt to find the area of parallelogram *PQRS* = $PR\times OQ=\left(8+"\frac{20}{3}"\right)\times"5"$ using their *P* and their *Q* and proceeding to find a value for the area. Condone attempts which appear to be calculating parallelogram *PQSR*.

Candidates may use the "Shoelace" algorithm i.e. $2\times\frac{1}{2}\begin{vmatrix}8 & 0\\ 0 & 5\\ -\frac{20}{3} & 0\\ 8 & 0\end{vmatrix}=2\times\frac{1}{2}\times\left(8\times5--\frac{20}{3}\times5\right)$

Candidates may attempt to find the area of the parallelogram using the determinant

eg $\frac{20}{3}\underset{\sim}{i}+5\underset{\sim}{j}$ and $8\underset{\sim}{i}-5\underset{\sim}{j}$ Area $=\left|\det\begin{bmatrix}\frac{20}{3} & 5\\ 8 & -5\end{bmatrix}\right|=\left|\frac{20}{3}\times-5-8\times5\right|=\left|-\frac{220}{3}\right|\Rightarrow$ Area $=\frac{220}{3}$

A1 $\frac{220}{3}$ oe (condone awrt 73.3)

(SC Sign slip on the coordinate *Q* to give $P=\left(-\frac{20}{3},0\right), Q=(0,-5)\Rightarrow$ Area of parallelogram = $\frac{220}{3}$ scores 011)

Some candidates who do part c) and form parallelogram *PQSR* to find $S=\left(\frac{44}{3},5\right)$ may then attempt b) and proceed to the correct answer. This can still score M1 A1 in (b)

(c)

M1 A full attempt to find both the coordinates of *S*. Examples include:

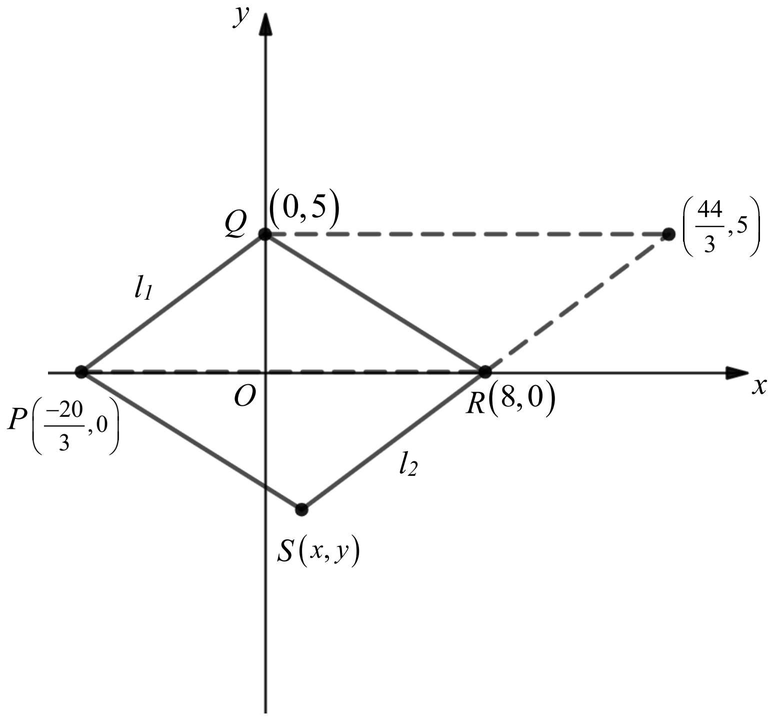
- $\overrightarrow{OP}+\overrightarrow{QR}$ or equivalent using a vector approach.
- An attempt to solve their $y=\frac{3}{4}x-6$ with their $y=-\frac{5}{8}\left(x+\frac{20}{3}\right)$
- An attempt to equate the length *RS* with *PQ* (or *PS* with *RQ*) to produce an equation that is solved simultaneously with l_2 or with each other (see next page)
- $y_S=-5$ from rotational symmetry consideration and proceeds to finding x
- It is also acceptable to score full marks following a sketch. Send to review if you find any which you think are credit-worthy.

In all cases they should proceed to find the coordinates of *S* but condone arithmetical slips in their working. If in doubt send to review.

A1 $\left(\frac{4}{3},-5\right)$. Allow alternatives eg $x=\ldots, y=\ldots$ Condone (awrt 1.33,5)

SC: For those students who find $S=\left(\frac{44}{3},5\right)$ score SC 10

Additional guidance: some examples of methods for (c) using the lengths between points



1. Equating *RS*: $(8,0)$ to (x,y) with *PQ:* $\left(\frac{-20}{3},0\right)$ to $(0,5)$ and solving simultaneously with l_2

Look for $(x-8)^2+y^2=\frac{625}{9}$ and $3x-4y-24=0$ proceeding to $x=\frac{4}{3},\ y=-5$

2. Equating *PS*: $\left(\frac{-20}{3},0\right)$ to (x,y) with *RQ:* $(8,0)$ to $(0,5)$ and solving simultaneously with l_2

Look for $(x--\frac{20}{3})^2+y^2=89$ and $3x-4y-24=0$ proceeding to $x=\frac{4}{3},\ y=-5$

3. Equating *RS*: $(8,0)$ to (x,y) with *PQ:* $\left(\frac{-20}{3},0\right)$ to $(0,5)$ and equating *PS*: $\left(\frac{-20}{3},0\right)$ to (x,y) with *RQ:* $(8,0)$ to $(0,5)$ and solving simultaneously

Look for $(x-8)^2+y^2=\frac{625}{9}$ and $(x--\frac{20}{3})^2+y^2=89$ which proceeds to

$9x^2-144x+9y^2-49=0$ and $9x^2+120x+9y^2-401=0 \Rightarrow 264x=352 \Rightarrow x=\frac{4}{3},\ y=-5$

Question Number	Scheme	Marks
7. (a)(i)	$P=(90°,3)$	M1, A1
(ii)	$Q=(540°,0)$	B1
		(3)
(b)	$(270°,4)$	M1, A1
		(2) **(5 marks)**

Note we condone the absence of the degree symbol in this question.

(a)(i)

M1 For one value in the coordinate pair of $(90°,3)$. Condone lack of brackets and do not be concerned with x or y correctly paired for this mark eg condone (…,90˚) or (3,…) Allow in radians.

A1 For both of $(90°,3)$. Allow $x=90,\ y=3$

(a)(ii)

B1 $Q=(540°,0)$ Allow $x=540,\ y=0$

(b)

M1 For one value in the coordinate pair of $(270°,4)$. Condone lack of brackets and do not be concerned with x or y correctly paired for this mark eg condone (…,270˚) or (4,…). Allow in radians.

A1 For both of $(270°,4)$. Allow $x=270,\ y=4$

For answers missing the brackets or radians(or both) in all parts then penalise the first time the A or B mark is due

FYI (a)(i) $\left(\frac{\pi}{2},3\right)$ (a)(ii) $Q=(3\pi,0)$, (b) $\left(\frac{3\pi}{2},4\right)$ would score M1 A0(first error) B1 M1 A1

For all correct values the wrong way round $(3,90°),(0,540°)\ (4,270°)$ SC M1A0B0M1A1

Question Number	Scheme	Marks
8.	Equates $y=k(2x-1)$ and $y=x^2+2x+11 \Rightarrow k(2x-1)=x^2+2x+11$	M1
	$\Rightarrow x^2+(2-2k)x+11+k \quad (=0)$	A1
	Attempts $"b^2-4ac"...0 \Rightarrow (2-2k)^2-4(11+k)...0$ and proceeds to critical values	M1
	Critical values of $(k=)\ 5, -2$	A1
	No roots so $b^2-4ac<0$ so choose inside region	M1
	$-2<k<5$	A1
		(6) **(6 marks)**

M1 Attempts to equate $y=k(2x-1)$ and $y=x^2+2x+11 \Rightarrow k(2x-1)=x^2+2x+11$

A1 $x^2+(2-2k)x+11+k \quad (=0)$ oe

Correct quadratic or correct values of *a*, *b* and *c*. The terms should be collected on one side of the equation and the *x* terms must have been collected together in a bracket. Condone a missing "=0". This mark may be implied by values of *a*, *b* and *c* used in later work.

M1 Attempts $"b^2-4ac"$ where $a=\pm 1,\ b=\pm 2\pm 2k,\ c=\pm 11\pm k$ to achieve a 3TQ and proceeds to find critical values. Condone arithmetical slips in the rearrangement to a 3TQ. Usual rules for solving a quadratic apply. You may see $"b^2...4ac"$.
FYI $"b^2-4ac"=4k^2-12k-40$ (Be careful that this may result from incorrect working which loses the A marks that follow)

A1 Correct critical values $(k=)\ 5, -2$ **MUST HAVE COME FROM CORRECT WORKING**

M1 Finds inside region for their critical values. May be awarded for eg $-2 \leqslant k \leqslant 5$ or $-2 \leqslant k < 5$. Allow this mark if another variable is used.

A1 $-2<k<5$ oe such as $k \in (-2,5)$ cso
Note that $-2<x<5$ is A0

Question Number	Scheme	Marks
9.	$\frac{4x^2+9}{2\sqrt{x}}=\frac{4x^2}{2\sqrt{x}}+\frac{9}{2\sqrt{x}}=2x^{\frac{3}{2}}+\frac{9}{2}x^{-\frac{1}{2}}$	M1 A1
	$\left(\frac{dy}{dx}=\right)\ 3x^{\frac{1}{2}}-\frac{9}{4}x^{-\frac{3}{2}}$	M1 A1
	$\left(\frac{dy}{dx}=\right)\ 3x^{\frac{1}{2}}-\frac{9}{4}x^{-\frac{3}{2}}=0\Rightarrow x^2=\frac{3}{4}\Rightarrow x=\frac{\sqrt{3}}{2}$	M1 A1
		(6) **(6 marks)**
Alt(I)	Quotient rule	
	$u=4x^2+9,u'=8x,v=2\sqrt{x},v'=x^{-\frac{1}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x-(4x^2+9)\times x^{-\frac{1}{2}}}{4x}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x-(4x^2+9)\times x^{-\frac{1}{2}}}{4x}=0\Rightarrow\ x^2=\frac{3}{4}\Rightarrow x=\frac{\sqrt{3}}{2}$	M1A1
Alt(II)	Product rule	
	$u=4x^2+9,u'=8x,v=\frac{1}{2}x^{-\frac{1}{2}},v'=\frac{1}{4}x^{-\frac{3}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ (4x^2+9)\times\frac{1}{4}x^{-\frac{3}{2}}+8x\times\frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x-(4x^2+9)\times x^{-\frac{1}{2}}}{4x}=0\Rightarrow\ x^2=\frac{3}{4}\Rightarrow x=\frac{\sqrt{3}}{2}$	M1A1

M1 Attempts to divide by $2\sqrt{x}$. Award for one correct term (including $\frac{9}{2x^{\frac{1}{2}}}$). Allow if they combine the two terms with a common denominator of 2, but the indices must have been processed.

If they use the quotient rule it is for selecting *u* and *v* and attempting to differentiate. Look for $u = 4x^2 + 9, u' = ...x, v = 2\sqrt{x}, v' = ...x^{-\frac{1}{2}}$. If they use the product rule then it is for $u = 4x^2 + 9, u' = 8x, v = \frac{1}{2}x^{-\frac{1}{2}}, v' = -\frac{1}{4}x^{-\frac{3}{2}}$

A1 $2x^{\frac{3}{2}} + \frac{9}{2}x^{-\frac{1}{2}}$ which may be left unsimplified but the indices must be processed. Using the product or quotient rule it is for having correct *u'* and *v'*.

M1 Attempts to differentiate the expression written as a sum. Award for one power decreasing by one on one of their terms following through their sum and the indices must have been processed. Cannot be awarded if they just differentiate top and bottom of the fraction.

Using the quotient rule look for expressions of the form $\left(\frac{dy}{dx} =\right) \frac{2\sqrt{x} \times ...x \pm (4x^2 + 9) \times ...x^{-\frac{1}{2}}}{4x}$

Using the product rule look for expressions of the form $\left(\frac{dy}{dx} =\right) (4x^2 + 9) \times ...x^{-\frac{3}{2}} + ...x \times \frac{1}{2}x^{-\frac{1}{2}}$

A1 $\left(\frac{dy}{dx} =\right) 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}}$ which may be left unsimplified but the indices must be processed.

Using the quotient rule award for $\left(\frac{dy}{dx} =\right) \frac{2\sqrt{x} \times 8x - (4x^2 + 9) \times x^{-\frac{1}{2}}}{4x}$.

Using the product rule award for $\left(\frac{dy}{dx} =\right) (4x^2 + 9) \times -\frac{1}{4}x^{-\frac{3}{2}} + 8x \times \frac{1}{2}x^{-\frac{1}{2}}$

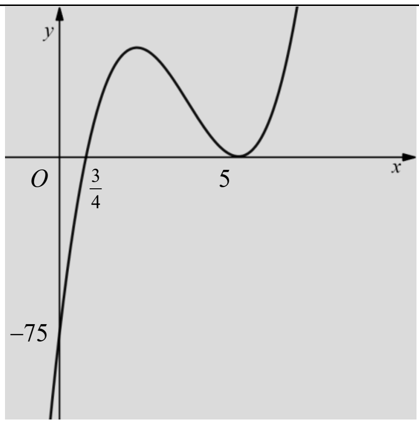
M1 Sets $\frac{dy}{dx} = 0$ and proceeds to $x^{\pm 2} = ...$ or $x^{\pm 4} = ...$ following a derivative in the form $\left(\frac{dy}{dx} =\right) Ax^{\frac{1}{2}} - Bx^{-\frac{3}{2}}, \quad A, B > 0$ (which may be unsimplified).

Using the quotient rule or product rule look to proceed from one of the forms above to $x^{\pm 2} = ...$ or $x^{\pm 4} = ...$

Rounded versions of the answer do not imply this mark. Do not be too concerned by the mechanics of their arrangement but we must see some attempt to carry out algebraic manipulation proceeding to the required form.

A1 $(x =)\ \frac{\sqrt{3}}{2}$ or exact equivalent cso (Note that a correct exact answer can imply the final M1A1 but a rounded answer with no working such as awrt 0.87 is M0A0)

Withhold the final mark if $-\frac{\sqrt{3}}{2}$ is not rejected.

Question Number	Scheme		Marks
10.(a)		Shape for +ve x^3	B1
		Cuts x-axis at $\left(\frac{3}{4},0\right)$ and meets at $(5,0)$	B1
		Crosses y-axis at $(0,-75)$	B1
			(3)
(b)(i)	$(x=)$ 3, 20		B1ft
(ii)	$(p=)75$		B1ft
			(2)
(c) (i)	$(\mathrm{g}(x)=)\quad (4(x+1)-3)(x+1-5)^2=(4x+1)(x-4)^2$		M1 A1
(ii)	16		B1
			(3)
			(8 marks)

(a)

For any of the coordinates if there are contradictions between the graph and the text then the graph takes precedence.

B1 Shape for a positive cubic, any position, with one maximum and one minimum. Condone no axes for this mark. Condone cubic curves which have a cusp like appearance at the minimum point.

B1 For a graph crossing at $\left(\frac{3}{4},0\right)$ and meeting at $(5,0)$. The graph should not stop or cross at $(5,0)$.

Allow just the x values instead of the full coordinates marked on the axes or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. **Only allow this mark if a graph is drawn.**

B1 For a graph crossing the y-axis at $(0,-75)$

Allow the y value to be marked instead of the full coordinate or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. **Only allow this mark if a graph is drawn.** Do not condone 75 marked on the negative y-axis.

Note: in both (b)(i) and (ii) for the follow through the graph takes precedence.
(b)(i)

B1ft $(x=)$ $3,\ 20$ and no others Follow through on $4\times$ their x intercepts.
Allow $(3,0)$ $(20,0)$ ignore $(0,-75)$

(b)(ii)

B1ft $(p=)$ 75 Follow through on their y intercept. Allow if $(y=)$ $\mathrm{f}(x)+75$ seen. Do not allow $y=75$

Mark (i) and (ii) together

(c)(i)

M1 Attempts $\mathrm{g}(x)=\left(4\left(x+1\right)-3\right)(x+1-5)^2$ condoning slips. Either award for sight of $x+1$ embedded in the equation or award for sight of an expression of the form $\left(4x+a\right)(x-4)^2$
Alternatively they expand $(4x-3)(x-5)^2=4x^3+Px^2+Qx+R$, where P,Q and $R\neq 0$, and then replace x with $x+1$ to achieve $4(x+1)^3+P(x+1)^2+Q(x+1)+R$ (condoning slips).

A1 $(\mathrm{g}(x)=)$ $\left(4x+1\right)(x-4)^2$ or simplified equivalent so accept $4x^3-31x^2+56x+16$

(c)(ii)
B1 16 but accept $(0,16)$.
Note that they may attempt $\mathrm{f}(1)=\left(4\times 1-3\right)(1-5)^2=16$

Question Number	Scheme	Marks
11 (a)	Gradient of normal $=\dfrac{1}{4}$	B1
	Equation of normal $(y+50)="\dfrac{1}{4}"(x-4)\Rightarrow y=\dfrac{1}{4}x-51$	M1 A1
		(3)
(b)	$\left(\mathrm{f}''(x)=\right)\ \dfrac{6}{\sqrt{x^3}}+x=6x^{-\frac{3}{2}}+x\Rightarrow \mathrm{f}'(x)=-12x^{-\frac{1}{2}}+\dfrac{1}{2}x^2+k$	M1 A1
	Substitutes $x=4,\ \mathrm{f}'(x)=-4\Rightarrow k=-6$	dM1 A1
	$\left(\mathrm{f}'(x)=\right)\ -12x^{-\frac{1}{2}}+\dfrac{1}{2}x^2-6\Rightarrow\left(\mathrm{f}(x)=\right)\ -24x^{\frac{1}{2}}+\dfrac{1}{6}x^3-6x+d$	dM1 A1ft
	Substitutes $x=4,\ \mathrm{f}(x)=-50\Rightarrow d=\dfrac{34}{3}$	dddM1
	$\left(\mathrm{f}(x)=\right)-24x^{\frac{1}{2}}+\dfrac{1}{6}x^3-6x+\dfrac{34}{3}$	A1
		(8) **(11 marks)**

Mark (a) and (b) together

(a)

B1 Deduces that the gradient of the normal is $\dfrac{1}{4}$

M1 Attempts to find the equation of a line passing through $P(4,-50)$ with a **changed** gradient. Allow one sign slip on a coordinate so either $(y+50)$ or $(x-4)$ must be correct. If they use $y=mx+c$ then at least one of the coordinates must be correctly substituted in and they must proceed as far as $c=...$

A1 $y=\dfrac{1}{4}x-51$

(b)

M1 Attempts to integrate $\dfrac{6}{\sqrt{x^3}}+x$ with one index correct. Either $...x^{-\frac{1}{2}}$ or $...x^2$

A1 $\left(\mathrm{f}'(x)\right)=-12x^{-\frac{1}{2}}+\dfrac{1}{2}x^2+k$ (unsimplified) with or without the $+k$

dM1 Substitutes $x=4,\ \mathrm{f}'(x)=-4$ into an integrated form (with + k) and proceeds to find the value of k This is dependent on the first M1.

A1 $\left(\mathrm{f}'(x)=\right)-12x^{-\frac{1}{2}}+\dfrac{1}{2}x^2-6$ (unsimplified) which may be implied

dM1 Dependent upon the first M. It is for integrating 'again' with one index correct. Either $...x^{\frac{1}{2}}$ or $...x^3$

A1ft $\left(\mathrm{f}(x)=\right)-24x^{\frac{1}{2}}+\dfrac{1}{6}x^3-6x+d$ (unsimplified) following through ONLY on their k (allow kx) and with or without d

dddM1 Dependent upon all three previous Ms.
Both "k" and "d" must have been added although condone calling them both c.
This mark is scored for using $x=4,\ \mathrm{f}(x)=-50$ in an attempt to find 'd'. Do not be concerned by the mechanics of their rearrangement.

A1 $\left(\mathrm{f}(x)=\right)\ -24x^{\frac{1}{2}}+\frac{1}{6}x^{3}-6x+\frac{34}{3}$ or **exact equivalent expressions**. Eg Do not allow $\frac{1}{6}$ to be written as 0.167 but condone $-6x^{1}$. The indices must have been processed and the terms must all be on one line including $\frac{34}{3}$. isw after a correct expression.